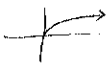
Math 70
Miscellaneous Final Exam Review Questions

Find the domain and range. Write your answers using interval notation.

1) $f(x)=\sqrt{x}$

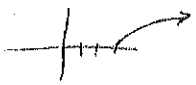


Domain: $[0,\infty)$

Range: $[0,\infty)$

2) $g(x)=\sqrt{x-3}$

$x-3\geq 0$
$x\geq 3$



Domain: $[3,\infty)$

Range: $[0,\infty)$

3) $h(x)=\sqrt{4-x}$

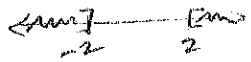
$4-x\geq 0$
$-x\geq -4$
$x\leq 4$

Domain: $(-\infty,4]$

Range: $[0,\infty)$

4) $g(x)=\sqrt{x^2-4}$

$x^2-4\geq 0$
$(x+2)(x-2)\geq 0$



Domain: $(-\infty,-2]\cup[2,\infty)$

Range: $[0,\infty)$

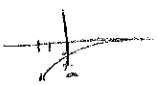
5) $f(x)=\sqrt{x}+1$

Domain: $[0,\infty)$

Range: $[1,\infty)$

6) $g(x)=\sqrt{x+2}-3$

$x+2\geq 0$
$x\geq -2$



Domain: $[-2,\infty)$

Range: $[-3,\infty)$

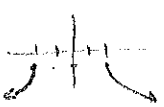
7) $h(x)=\sqrt{3-x}+2$

$3-x\geq 0$
$3\geq x$
$x\leq 3$

Domain $(-\infty,3]$

Range $[2,\infty)$

8) $g(x)=\sqrt{x^2-4}-1$



Domain: $(-\infty,-2]\cup[2,\infty)$

Range: $(-\infty,-1]$

Solve the equations.

9) $36x^3-25x=0$

GCF: $x(36x^2-25)=0$

Diff of Sq: $x(6x-5)(6x+5)=0$.

$\boxed{x=0 \quad x=\frac{5}{6} \quad x=-\frac{5}{6}}$

10) $27x^3-8=0$

Diff of cubes: $a^3-b^3=(a-b)(a^2+ab+b^2)$

$a=\sqrt[3]{27x^3}=3x$

$b=\sqrt[3]{8}=2$

$(3x-2)(9x^2+6x+4)=0$

$\boxed{x=\frac{2}{3}}$ $\quad x=\frac{-6\pm\sqrt{36-4(9)(4)}}{2(9)}$

$x=\frac{-6\pm\sqrt{-108}}{18}=-\frac{1}{3}\pm\frac{6\sqrt{3}}{18}=\boxed{-\frac{1}{3}+\frac{\sqrt{3}}{3}}$

11) $6x^2-x-35=0$

QF $x=\frac{-(-1)\pm\sqrt{(-1)^2-4(6)(-35)}}{2(6)}$

$=\frac{1\pm\sqrt{1+840}}{12}$

$=\frac{1\pm 29}{12}=\frac{30}{12},\frac{-28}{12}=\boxed{\frac{5}{2},\frac{-7}{3}}$

OR factor:

$(2x-5)(3x+7)=0$

$x=5/2 \quad x=-7/3$

12) $49x=100x^3$

set=0: $0=100x^3-49x$

GCF: $0=x(100x^2-49)$

diff of sq: $0=x(10x-7)(10x+7)$

$\boxed{x=0 \quad x=\frac{7}{10} \quad x=-\frac{7}{10}}$

13) $-1=64y^3$

set=0 $0=64y^3+1$

sum of cubes: $a^3+b^3=(a+b)(a^2-ab+b^2)$

$a=\sqrt[3]{64y^3}=4y$

$b=\sqrt[3]{1}=1$

$0=(4y+1)(16y^2-4y+1)$

$\boxed{y=-\frac{1}{4}}$ $\quad y=\frac{4\pm\sqrt{(-4)^2-4(16)(1)}}{2(16)}=\frac{1}{8}\pm\frac{\sqrt{-48}}{32}$

$=\frac{1}{8}\pm\frac{4\sqrt{3}}{32}$

$=\boxed{\frac{1}{8}\pm\frac{i\sqrt{3}}{8}}$

14) $35x^2-8x-3=0$

QF $x=\frac{8\pm\sqrt{(-8)^2-4(35)(-3)}}{2(35)}$

$=\frac{8\pm\sqrt{64+420}}{70}$

$=\frac{8\pm\sqrt{484}}{70}$

$=\frac{4}{35}\pm\frac{22}{70}$

$=\boxed{\frac{3}{7},-\frac{1}{5}}$

OR factor:

$(7x-3)(5x+1)=0$

$\boxed{x=3/7 \quad x=-1/5}$

Solve the systems of linear equations using matrices and identify the solution for the z variable. Give an exact answer.

15) $x+y+z=-1.6$
$2x-y-3z=7.2$
$2y+5z=-6$

$\rightarrow \begin{bmatrix} 1 & 1 & 1 & -1.6 \\ 2 & -1 & -3 & 7.2 \\ 2 & 2 & 5 & -6 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 56/45 \\ 0 & 1 & 0 & -86/45 \\ 0 & 0 & 1 & -14/15 \end{bmatrix}$ $\boxed{z=-14/15}$

16) $5x-3y+z=-12$
$x+2z=-3.75$
$y-z=3.75$

$\rightarrow \begin{bmatrix} 5 & -3 & 1 & -12 \\ 1 & 0 & 2 & -3.75 \\ 0 & 1 & -1 & 3.75 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 & 0 & 0 & -.75 \\ 0 & 1 & 0 & 2.25 \\ 0 & 0 & 1 & -1.5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -3/4 \\ 0 & 1 & 0 & 9/4 \\ 0 & 0 & 1 & -3/2 \end{bmatrix}$ $\boxed{z=-3/2}$

17) $4x+y-z=\frac{5}{3}$
$x-3y+z=\frac{37}{3}$
$2x+5z=\frac{8}{3}$

$\rightarrow \begin{bmatrix} 4 & 1 & -1 & 5/3 \\ 1 & -3 & 1 & 37/3 \\ 2 & 0 & 5 & 8/3 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 & 0 & 0 & 4/3 \\ 0 & 1 & 0 & -11/3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $\boxed{z=0}$

) D frac
...roughout

Solve for the specified variable.

18) $\frac{2a}{3a-b}=c$ for a

cross-multiply
$2a=c(3a-b)$
dist
$2a=3ac-cb$
collect a's
$cb=3ac-2a$
factor out a
$cb=a(3c-2)$
isolate a
$\boxed{a=\frac{cb}{3c-2}}$

19) $\frac{b+2c}{b-c}=4$ for b

$b+2c=4(b-c)$
$b+2c=4b-4c$
$2c+4c=4b-b$
$6c=3b$
$\frac{6c}{3}=b$
$\boxed{2c=b}$

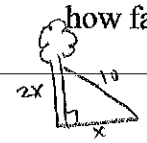
20) $\frac{2d+f}{g+h}=\frac{x+c}{3x}$ for x.

Factor fractional answers.

$3x(2d+f)=(x+c)(g+h)$
$6dx+3fx=gx+hx+cg+ch$
$6dx+3fx-gx-hx=cg+ch$
$x(6d+3f-g-h)=cg+ch$
$\boxed{x=\frac{c(g+h)}{6d+3f-g-h}}$

Solve the problem.

32) A tree stands straight when it is supported by a 10-foot rope which is tied to the tree and staked to the ground x feet away from the base of the tree. If the rope is tied to the tree trunk 2x feet above the ground, how far is the stake from the tree? Hint: Pythagorean Theorem.




$x^2+(2x)^2=10^2$
$x^2+4x^2=100$
$5x^2=100$
$x^2=20$
$x=\pm\sqrt{20}$
$\boxed{x=+2\sqrt{5}\text{ ft}}$

If it says round to 4 places:
$x\approx 4.4721$ ft

$\boxed{M=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)}$ Midpt formula

Find the midpoint of the line segment between:

21) (5, -3) and (-2, 8)
$\left(\frac{5+-2}{2},\frac{-3+8}{2}\right)=\boxed{\left(\frac{3}{2},\frac{5}{2}\right)}$

22) (7,10) and (7,-3)
$\left(\frac{7+7}{2},\frac{10+(-3)}{2}\right)=\boxed{\left(7,\frac{7}{2}\right)}$

23) (14,0) and (-2, -6)
$\left(\frac{14+(-2)}{2},\frac{0-6}{2}\right)=\boxed{(6,-3)}$

24) (-3,-9) and (1,5)
$\left(\frac{-3+1}{2},\frac{-9+5}{2}\right)=\boxed{(-1,-2)}$

Find the distance between the points.

$\boxed{D=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}}$ Distance Formula

25) (5, -3) and (-2, 8)
$\sqrt{(5-(-2))^2+(-3-8)^2}=\sqrt{49+121}=\boxed{\sqrt{170}}$
170 = 2 · 85 = 5 · 17

26) (7,10) and (7,-3)
$\sqrt{(7-7)^2+(10-3)^2}=\sqrt{0+169}=\boxed{13}$

27) (14,0) and (-2, -6)
$\sqrt{(14--2)^2+(0-6)^2}=\sqrt{256+36}=\sqrt{292}=\boxed{2\sqrt{73}}$
292 = 4 · 73 = 2 · 2 · 73

28) (-3,-9) and (1,5)
$\sqrt{(-3-1)^2+(-9-5)^2}=\sqrt{16+2025}=\boxed{\sqrt{2041}}$
2041 = 13 · 157

Write the equation of a parabola having the same shape (and direction) as

29) $f(x)=-2x^2$, but with vertex (3, -4)
$\boxed{y=-2(x-3)^2-4}$

30) $f(x)=4x^2+1$, but with vertex (-6, 9)
$\boxed{y=4(x+6)^2+9}$

31) $h(x)=2(x-3)^2+5$, but with vertex (1, -7)
$\boxed{y=2(x-1)^2-7}$
x coord of vertex; y coord of vertex